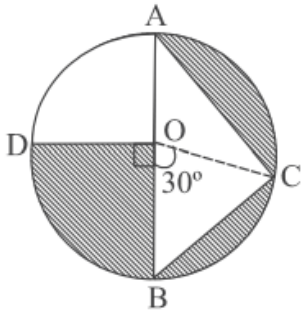
Area Related to Circles

1. O is the centre of the circle. If AC = 28 cm, BC= 21 cm, <BOD = 90° and <BOC = 30°, then find the area of the shaded region given in the figure. (2024)



Answer. Assuming AOB to be a straight line and hence the diameter of the circle. $\Rightarrow \angle$ ACB = 90°

Then in Δ ACB, $AC^2 + BC^2 = 28^2 + 21^2 = (35)2 = AB^2$

$\therefore$ AB = 35 cm is the diameter and $\Rightarrow$ $r = \frac{35}{2}$ cm

Area of shaded region

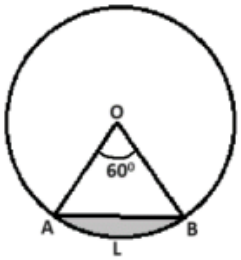
= area of quadrant + $(\frac{1}{2} \times \pi r^2$ – area of Δ ACB)

$$= \left(\frac{3}{4} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2}\right) - \frac{1}{2} \times 28 \times 21$$

= 721·9 – 294 = 427·9 (approx)

2. An arc of a circle of radius 21 cm subtends an angle of 60° at the centre. Find:
(i) the length of the arc.
(ii) the area of the minor segment of the circle made by the corresponding chord. (2024)

Answer.



(i) Length of the arc AB $= 2 \times \frac{22}{7} \times 21 \times \frac{60}{360}$

$= 22$ cm

(ii) Area of sector OALB $= \frac{22}{7} \times 21 \times 21 \times \frac{60}{360} = 231$ cm^2

Area of $\Delta OAB = \frac{\sqrt{3}}{4} \times 21 \times 21 = \frac{441\sqrt{3}}{4}$ cm^2

Area of minor segment $= \left(231 - \frac{441\sqrt{3}}{4}\right)$ cm^2

or $(231 - 190.95) = 40.05$ cm^2

Perimeter and Area of a Circle-A Review

MCQ

1. What is the area of a semi-circle of diameter 'd'?

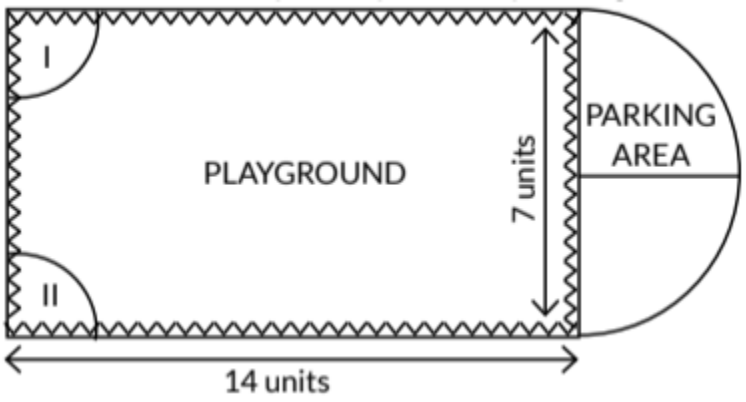
(a) $\frac{1}{16}\pi d^2$ (b) $\frac{1}{4}\pi d^2$ (c) $\frac{1}{8}\pi d^2$ (d) $\frac{1}{2}\pi d^2$

(2023)

2. In a right triangle ABC, right-angled at B, BC = 12 cm and AB = 5 cm. The radius of the circle inscribed in the triangle (in cm) is
(b) 3
(a) 4
(c) 2
(d) 1 (Al 2014) Ap

LA (4/5/6 marks)

3. Case Study: Governing council of a local public development authority of Dehradun decided to build an adventurous playground on the top of a hill, which will have adequate space for parking.



After survey, it was decided to build rectangular playground, with a semi-circular area allotted for parking at one end of the playground. The length and

breadth of the rectangular playground are 14 units and 7 units, respectively. There are two quadrants of radius 2 units on one side for special seats. Based on the above information, answer the following questions:
(i) What is the total perimeter of the parking area?
(ii) (a) What is the total area of parking and the two quadrants?

OR

(b) What is the ratio of area of playground to the area of parking area?
(iii) Find the cost of fencing the playground and parking area at the rate of 2 per unit. (2023)

11.1 Areas of Sector and Segment of a Circle

MCQ

4. The area swept by 7 cm long minute hand of a clock in 10 minutes is

(a) $77\ cm^2$ (b) $12\frac{5}{6} cm^2$

(c) $7\frac{1}{12} cm^2$ (d) $25\frac{2}{3} cm^2$

(Term I, 2021-22)

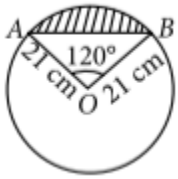
SAI (2 marks)

5. A piece of wire 22 cm long is bent into the form of an arc of a circle subtending an angle of 60° at its centre. Find the radius of the circle.

SA II (3 marks)

6. A car has two wipers which do not overlap. Each wiper has a blade of length 21 cm sweeping through an angle 120°. Find the total area cleaned at each sweep of the blades. (Take $\pi = \frac{22}{7}$) *(2019)*

7. Find the area of the segment shown in the given figure, if radius of the circle is 21 cm and $\angle AOB = 120°$



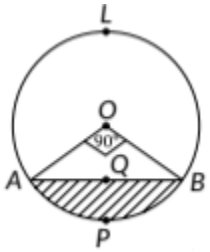
$\left(\text{Use } \pi = \frac{22}{7}\right)$ *(Delhi 2019)*

8 In the given figure, three sectors of a circle of radius 7 cm, making angles of 60°, 80° and 40° at the centre are shaded. Find the area of the shaded region.

(AI 2019) An

9. In the given figure, AB is a chord of a circle, with centre O and radius 10 cm, that subtends a right angle at the centre of the circle. Find the area of the minor segment AQBP. Hence, find the area of major segment ALBQA.

[Use π = 3.14] *(Foreign 2016)*



10. Find the area of the minor segment of a circle of radius 14 cm, when its central angle is 60°. Also find the area of the corresponding major segment.

$\left[\text{Use } \pi = \frac{22}{7}\right]$ *(AI 2015)*

LA (4/5/6 marks)

11. Achord of a circle of radius 14 cm subtends an angle of 60° at the centre. Find the area of the corresponding minor segment of the circle. Also find the area of the major segment of the circle. (2023)

12. A chord PQ of a circle of radius 10 cm subtends an angle of 60° at the centre of circle. Find the area of major and minor segments of the circle. (Delhi 2017)

CBSE Sample Questions

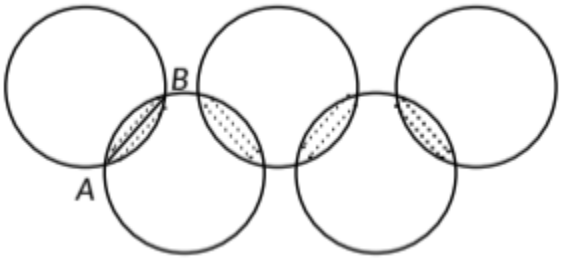
11.1 Areas of Sector and Segment of a Circle

MCQ

1. The area of the circle that can be inscribed in a square of 6 cm is
(a) $36л\ cm^2$
(b) $18л\ cm^2$
(c) $12\pi\ cm^2$
(d) $9\ cm^2$ (2022-23)

2. The number of revolutions made by a circular wheel of radius 0.25m in rolling a distance of 11km is
(a) 2800
(b) 4000
(c) 5500
(d) 7000 (2022-23)

3. Given below is the picture of the Olympic rings made by taking five congruent circles of radius 1 cm each, intersecting in such a way that the chord formed by joining the point of intersection of two circles is also of length 1 cm. Total area of all the dotted regions assuming the thickness of the rings to be negligible is



(a) $4\left(\frac{\pi}{12}-\frac{\sqrt{3}}{4}\right)cm^2$ (b) $\left(\frac{\pi}{6}-\frac{\sqrt{3}}{4}\right)cm^2$

(c) $4\left(\frac{\pi}{6}-\frac{\sqrt{3}}{4}\right)cm^2$ (d) $8\left(\frac{\pi}{6}-\frac{\sqrt{3}}{4}\right)cm^2$

(Term I, 2021-22

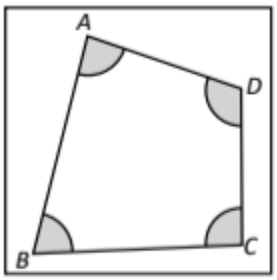
VSA (1 mark)

4. In a circle of diameter 42 cm, if an arc subtends an angle of 60° at the centre where $\pi = 22/7$, then what will be the length of arc? (2020-21)

SAI (2 marks)

5. The length of the minute hand of a clock is 6 cm. Find the area swept by it when it moves from 7:05 p.m. to 7:40 p.m. (2022-23)

6. In the given figure, arcs have been drawn of radius 7 cm each with vertices A, B, C and D of quadrilateral ABCD as centres. Find the area of the shaded region.



(2022-23) Ap

SOLUTIONS

Previous Years' CBSE Board Questions

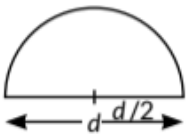
1.

(c) : Given, diameter of semi-circle = d

$\therefore$ Radius, $r = d/2$

$\therefore$ Area of semi circle $= \frac{1}{2}\pi\left(\frac{d}{2}\right)^2$

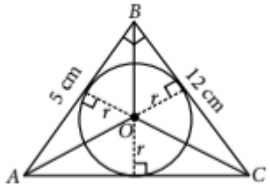
$= \frac{1}{2}\pi \times \frac{d^2}{4} = \frac{1}{8}\pi d^2$



2.

(c) : Given, ΔABC is a triangle right angled at B.

$\therefore$ By using Pythagoras theorem, $AC = 13$ cm



$$\text{Area of } \Delta ABC = \frac{1}{2} \times 5 \times 12 = 30 \text{ cm}^2$$

Also, Area of ΔABC = Area of ΔAOC + Area of ΔBOC + Area of ΔAOB

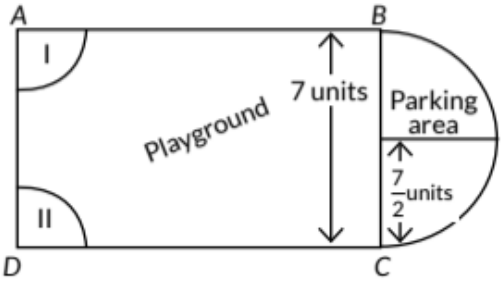
$$\Rightarrow 30 = \frac{1}{2} \times 13 \times r + \frac{1}{2} \times 12 \times r + \frac{1}{2} \times 5 \times r$$

[where r = radius of circle]

$$\Rightarrow 30 = \frac{13r}{2} + \frac{12r}{2} + \frac{5r}{2} \Rightarrow 30r = 60 \Rightarrow r = 2 \text{ cm}$$

Hence, radius of circle = 2 cm

3. (i) Length of play ground, AB = 14 units, Breadth of play ground, AD = 7 units
Radius of semi-circular part is 7/2 units Total perimeter of parking area = πr+2r



$$= \frac{22}{7} \times \frac{7}{2} + 2 \times \frac{7}{2} = 11 + 7 = 18 \text{ units}$$

(ii) (a) : Area of parking $= \frac{\pi r^2}{2}$

$$= \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

= 19.25 sq. units

Area of two quadrants (I) and (II) $= 2 \times \frac{1}{4} \times \pi r^2$

$$= \frac{1}{2} \times \frac{22}{7} \times 2 \times 2$$

= 6.29 sq. units.

Total area of parking and two quadrant

= 19.25 + 6.29 = 25.54 sq. units

OR

(b) Area of playground = length x breadth = 14x7 = 98 sq. units

Area of parking $= \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

Required ratio $= \frac{98}{\frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}} = \frac{98 \times 4}{77} = \frac{56}{11} = 56:11$

(iii) Perimeter of parking area = 18 units.

So, the cost of fencing the parking area = (18 x 2)=36

Length of remaining three sides of playground

= 14+14+7=35 units

Now, the cost of fencing three sides = √2 × 35 = *70

Total cost = 36+ 70 = 106

4. (d): Angle formed by minute hand of a clock in

60 minutes = 360°

.. Angle formed by minute hand of a clock in 10 minutes

$= \frac{10}{60} \times 360° = 60°$

Length of minute hand of a clock = radius = 7 cm

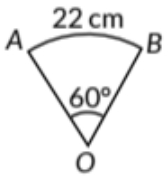
$\therefore$ Required area

$$= \pi r^2 \times \frac{\theta}{360°} = \frac{22}{7} \times 7 \times 7 \times \frac{60°}{360°} = \frac{77}{3}\text{cm}^2 = 25\frac{2}{3}\text{ cm}^2$$

5. Let AB be the wire of length 22 cm in the form of an arc of a circle subtending an ZAOB = 60° at centre O.

$\because$ Length of arc $= 2\pi r\left(\frac{\theta}{360°}\right)$

$\Rightarrow \quad 22 = 2 \times \frac{22}{7} \times r\left(\frac{60°}{360°}\right) \Rightarrow r = \frac{7 \times 6}{2} = 21\text{ cm}$



Hence, radius of the circle is 21 cm.

6. Here radius (r) = 21 cm

Sector angle (0) = 120°

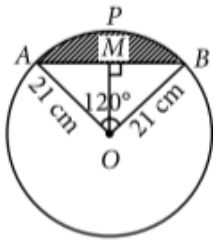
:- Area cleaned by each sweep of the blades

$= \left[\frac{\theta}{360°} \times \pi r^2\right] \times 2$ ($\because$ there are 2 blades)

$= \left[\frac{120°}{360°} \times \frac{22}{7} \times 21 \times 21\right] \times 2 = 22 \times 7 \times 3 \times 2\text{ cm}^2 = 924\text{ cm}^2$

7. Given, O is the centre of the circle of radius 21 cm and AB is the chord that subtends an angle of 120° at the centre.

Draw $OM \perp AB$.



Area of the minor segment $AMBP$

= Area of sector $OAPB$ – Area of ΔAOB

Now, area of sector $OAPB$

$$= \frac{\theta}{360^\circ} \times \pi r^2 = \frac{120^\circ}{360^\circ} \times \frac{22}{7} \times 21 \times 21 = 462\,\text{cm}^2$$

Since, $OM \perp AB$.

$$\angle AOM = \angle BOM = \frac{120^\circ}{2} = 60^\circ$$

[$\because$ Perpendicular from the centre to the chord bisects the angle subtended by the chord at the centre.]

$$\text{In } \Delta AOM, \sin 60^\circ = \frac{AM}{AO}, \cos 60^\circ = \frac{OM}{OA}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{AM}{21}, \frac{1}{2} = \frac{OM}{21} \Rightarrow \quad AM = \frac{21\sqrt{3}}{2}\,\text{cm}, OM = \frac{21}{2}\,\text{cm}$$

$$\because \quad AB = 2AM = 2 \times \frac{21\sqrt{3}}{2} = 21\sqrt{3}\,\text{cm}$$

$$\text{Area of } \Delta AOB = \frac{1}{2} \times AB \times OM = \frac{1}{2} \times 21\sqrt{3} \times \frac{21}{2} = \frac{441\sqrt{3}}{4}\,\text{cm}^2$$

$$\text{Hence, required area} = 462 - \frac{441\sqrt{3}}{2}$$

$$= 462 - 381.92 = 80.08\,\text{cm}^2$$

8.

Radius (r) of circle = 7 cm

$$\text{Area of shaded region} = \frac{\pi(7)^2 \cdot 40^\circ}{360^\circ} + \frac{\pi(7)^2 \cdot 60^\circ}{360^\circ} + \frac{\pi(7)^2 \cdot 80^\circ}{360^\circ}$$

$$\left[\because \text{ Area of sector } = \frac{\theta}{360^\circ}\pi r^2\right]$$

$$= \frac{\pi(7)^2}{9} + \frac{\pi(7)^2}{6} + \frac{\pi(7)^2 \cdot 2}{9} = \pi(7)^2\left[\frac{1}{9} + \frac{1}{6} + \frac{2}{9}\right]$$

$$= \frac{22}{7} \times 7 \times 7 \times \frac{9}{18} = 77\,\text{cm}^2$$

9.

We have, radius (r) = 10 cm and $\theta = 90°$

So, area of sector $OAPB = \frac{\theta}{360°}\pi r^2$

$= \frac{90°}{360°} \times 3.14 \times 10^2 = 78.5\ cm^2$

Area of $\Delta OAB = \frac{1}{2} \times 10 \times 10 = 50\ cm^2$

$\therefore$ Area of the minor segment $AQBP$ = Area of sector $OAPB$ – Area of ΔOAB = (78.5 – 50) cm = 28.5 cm^2

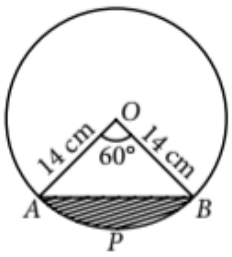
Area of circle = $\pi r^2 = 3.14 \times 10^2 = 314\ cm^2$

$\therefore$ Area of major segment $ALBQA$

= Area of circle – Area of minor segment $AQBP$

= (314 – 28.5) cm^2 = 285.5 cm^2

10. We have, radius (r) = 14 cm and 0 = 60° Area of minor segment



= Area of sector $OAPB$ – Area of ΔOAB

$= \frac{\theta \pi r^2}{360°} - \frac{1}{2} r^2 \sin\theta$

$= \frac{60° \times 22 \times 14 \times 14}{7 \times 360°} - \frac{1}{2} \times 14 \times 14 \times \sin 60°$

$= \frac{22 \times 14}{3} - 7 \times 14 \times \frac{\sqrt{3}}{2} = 102.67 - 84.87 = 17.8\ cm^2$

Area of circle = $\pi r^2 = \frac{22}{7} \times 14 \times 14 = 616\ cm^2$

Area of major segment = Area of circle - Area of minor segment

= (616-17.8) cm^2 = 598.2 cm^2

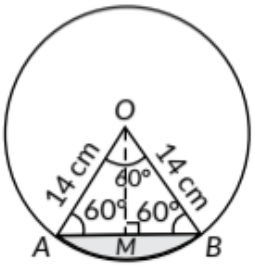
11.

Here, radius (r) = 14 cm and
Sector angle (θ) = 60°

$\therefore$ Area of the sector

$$= \frac{\theta}{360^\circ} \times \pi r^2 = \left(\frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14 \right) \text{cm}^2$$

$$= 102.67 \text{ cm}^2$$



Since $\angle O = 60^\circ$ and $OA = OB = 14$ cm

$\therefore$ AOB is an equilateral triangle.

$\Rightarrow$ $AB = 14$ cm and $\angle A = 60^\circ$

Draw $OM \perp AB$,

In ΔAMO

$$\frac{OM}{OA} = \sin 60^\circ = \frac{\sqrt{3}}{2} \Rightarrow OM = OA \times \frac{\sqrt{3}}{2} = \frac{14\sqrt{3}}{2} \text{cm} = 7\sqrt{3} \text{ cm}$$

Now, $ar(\Delta AOB) = \frac{1}{2} \times AB \times OM$

$$= \frac{1}{2} \times 14 \times 7\sqrt{3} \text{ cm}^2 = 49\sqrt{3} \text{ cm}^2$$

$$= 49 \times 1.732 \text{ cm}^2 = 84.87 \text{ cm}^2$$

Now, area of the minor segment

= (Area of minor sector) – ($ar\ \Delta AOB$)

= 102.67 – 84.87 cm^2 = 17.8 cm^2

Area of the major segment

= Area of the circle – Area of the minor segment

= (πr^2 – 17·8)

$$= \left[\left(\frac{22}{7} \times 14 \times 14 \right) - 17 \cdot 8 \right] \text{cm}^2$$

= (616 – 17.8) cm^2 = 598.2 cm^2

12. We have, radius (r) = 10 cm and 0 = 60°

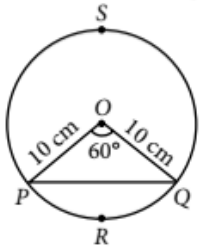
Area of minor segment PQR = Area of sector OPRQ

$= \frac{\theta}{360°} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$

$= \frac{60°}{360°} \times \frac{22}{7} \times 10 \times 10 - \frac{1}{2} \times 10 \times 10 \times \sin 60°$

$= \frac{1100}{21} - 25\sqrt{3} = 52.38 - 43.3 = 9.08\ \text{cm}^2$

– Area of ΔOPQ



Area of major segment PSQ = Area of circle – Area of minor segment

$= \pi(10)^2 - 9.08 = 314.28 - 9.08 = 305.2\ \text{cm}^2$

CBSE Sample Questions

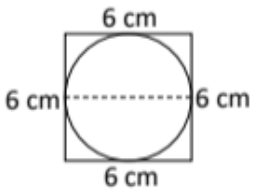
1.

(d): Diameter of circle can be 6 cm

then radius (r) = 3 cm

Area of circle is ; $A = \pi r^2$

$= \pi (3)^2 = 9\pi\ \text{cm}^2$ **(1)**



2. (d): In one revolution wheel covers distance of 2πr. So, in n revolution it will cover 2πrn distance.

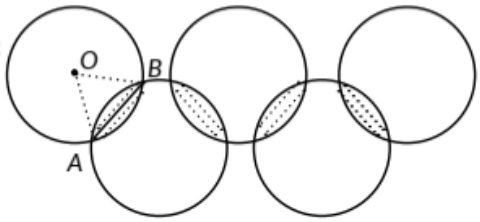
:- S = 2πrn

According to question, S = 11 km, r = 0.25 m so,

$11 \times 1000 = n \times 2 \times \frac{22}{7} \times 0.25 \Rightarrow n = 7000$ **(1)**

3.

(d):



Let O be the centre of the circle. So, $OA = OB = AB = 1$ cm

So ΔOAB is an equilateral triangle. $\therefore\ \angle AOB = 60°$

$\therefore$ Required area = 8 × area of one segment with r = 1 cm, $\theta = 60°$

= 8 × {area of sector – area of ΔAOB}

$= 8 \times \left(\frac{60°}{360°} \times \pi \times 1^2 - \frac{\sqrt{3}}{4} \times 1^2 \right) = 8\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4} \right) \text{cm}^2$ **(1)**

4.

$$\text{Given } \theta = 60°, r = \frac{42}{2} = 21 \text{ cm}$$

$$\text{So, length of arc} = 2\pi r\left(\frac{\theta}{360°}\right) \quad \textbf{(1/2)}$$

$$= 2\times\frac{22}{7}\times 21\times\frac{60°}{360°} = 22 \text{ cm} \quad \textbf{(1/2)}$$

5. We know that, in 60 minutes, the tip of minute hand moves 360°. In 1 minute, it will move = 360°/60 = 6°
:- From 7:05 pm to 7:40 pm i.e. 35 min, it will move through = 35 × 6° = 210° (1)
:- Area swept by the minute hand in 35 min = Area of sector with sectorial angle 0 of 210° and radius of 6 cm

$$=\frac{210°}{360°}\times\pi\times 6^2 = \frac{7}{12}\times\frac{22}{7}\times 6\times 6 = 66 \text{ cm}^2 \quad \textbf{(1)}$$

6. Let the measure of ZA, ZB, ZC and D be 01, 02, 03 and 04 respectively
Required area = Area of sector with centre A + Area of sector with centre B + Area of sector with centre C + Area of sector with centre D

$$=\frac{\theta_1}{360°}\times\pi\times 7^2+\frac{\theta_2}{360°}\times\pi\times 7^2+\frac{\theta_3}{360°}\times\pi\times 7^2+\frac{\theta_4}{360°}\times\pi\times 7^2 \quad \textbf{(1)}$$

$$=\frac{(\theta_1+\theta_2+\theta_3+\theta_4)}{360°}\times\pi\times 7^2 = \frac{(360°)}{360°}\times\frac{22}{7}\times 7\times 7$$

(By angle sum property of a quadrilateral)

$$= 154 \text{ cm}^2 \quad \textbf{(1)}$$